How I arrived at Eagle 53.

This article describes how I arrived at my ultimate musical tuning: Eagle 53. Below is a list of just melodic intervals whose strength values are greater than 0.25 using the 2/x + 2/y formula over a one octave range. I used this list as a map when working out Eagle J.I. (J.I. is short for Just Intonation) Intervals with a value greater than 1.0 are high strength melodic intervals and intervals with a value between 0.25 and 0.999 are medium strength melodic intervals. Anything else is a low strength interval. The strength values are shown in the middle column. The numbers on the right are the widths of the intervals in cents.

1/1	4	0.0000
16/15	0.2583	111.7313
15/14	0.2762	119.4428
14/13	0.2967	128.2982
13/12	0.3205	138.5727
12/11	0.3485	150.6371
11/10	0.3818	165.0042
10/9	0.4222	182.4037
9/8	0.4722	203.9100
17/15	0.2510	216.6867
8/7	0.5357	231.1741
15/13	0.2872	247.7411
7/6	0.6190	266.8709
13/11	0.3357	289.2097
6/5	0.7333	315.6413
17/14	0.2605	336.1295
11/9	0.4040	347.4079
16/13	0.2788	359.4723
5/4	0.9000	386.3137
14/11	0.3247	417.5080
9/7	0.5079	435.0841
13/10	0.3538	454.2140
17/13	0.2715	464.4278
4/3	1.1667	498.0450
15/11	0.3152	536.9508
11/8	0.4318	551.3179
18/13	0.2650	563.3823
7/5	0.6857	582.5122
17/12	0.2843	603.0004
10/7	0.4857	617.4878
13/9	0.3761	636.6177
16/11	0.3068	648.6821
19/13	0.2591	656.9854
3/2	1.6667	701.9550
20/13	0.2538	745.7861
17/11	0.2995	753.6375
14/9	0.3651	764.9159
11/7	0.4675	782.4920
19/12	0.2719	795.5580
8/5	0.6500	813.6863
13/8	0.4038	840.5277
18/11	0.2929	852.5921

5/3	1.0667	884.3587
17/10	0.3176	918.6417
12/7	0.4524	933.1291
19/11	0.2871	946.1951
7/4	0.7857	968.8259
16/9	0.3472	996.0900
9/5	0.6222	1017.5963
20/11	0.2818	1034.9958
11/6	0.5152	1049.3629
13/7	0.4396	1071.7018
15/8	0.3833	1088.2687
17/9	0.3399	1101.0454
19/10	0.3053	1111.1993
21/11	0.2771	1119.4630
23/12	0.2536	1126.3193
2/1	3	1200.0000

First of all, like most tunings, I start with two notes (1/1 and 2/1, an octave apart) and then add some notes in between. My tuning will be *octave repeating* (meaning that the notes in any given octave range will have frequencies exactly double that of the notes in the octave just below it and the notes will have frequencies exactly half of the notes in the octave just above it).

I think that the melodic aspect of a tuning is more important than the harmony aspect. Having a lot of good chords available is not much good if you can’t play a decent melody. As I said I use the list of high strength and medium strength melodic intervals above as a *map* when I am working out a tuning.

At one point I wanted every note in the new tuning, when paired with every note (over a one octave range) to make a high or medium strength melodic interval within 6.7758 cents (256/255 my chosen tempering tolerance) accuracy of any interval in the list above. With a 12 note per octave tuning I don't think that this can be done.

I consider the tonics 1/1 and 2/1 to be more important than the notes in between and what *can* be done is to find 11 notes between 1/1 and 2/1 that are high or medium strength when paired melodically with *both* 1/1 *and* 2/1. Low strength intervals may occur elsewhere in the tuning but never in an interval that contains either the 1/1 or 2/1 notes over the one octave range.

Starting with 1/1 and 2/1 I add 4/3 and 3/2 to the tuning. Both 4/3 and 3/2 are high strength melodically when paired with both 1/1 and 2/1.

Next I add 5/3 because it is high strength melodically when paired with 1/1 and goes well with 2/1.

Now I add 6/5 because it is high strength melodically when paired with 2/1 and is also good (medium strength) when paired with 1/1. I now have...

1/1, 6/5, 4/3, 3/2, 5/3, 2/1.

Between 6/5 and 4/3 I have...

6/5	0.7333	315.6413
17/14	0.2605	336.1295
11/9	0.4040	347.4079
16/13	0.2788	359.4723
5/4	0.9000	386.3137
14/11	0.3247	417.5080
9/7	0.5079	435.0841
13/10	0.3538	454.2140
17/13	0.2715	464.4278
4/3	1.1667	498.0450

5/4 is the obvious choice as it is good melodically when paired with both 1/1 and 2/1 and is better than 9/7. So 5/4 is in.

Between 4/3 and 3/2 I have...

4/3	1.1667	498.0450
15/11	0.3152	536.9508
11/8	0.4318	551.3179
18/13	0.2650	563.3823
7/5	0.6857	582.5122
17/12	0.2843	603.0004
10/7	0.4857	617.4878
13/9	0.3761	636.6177
16/11	0.3068	648.6821
19/13	0.2591	656.9854
3/2	1.6667	701.9550

The likeliest candidates here are 7/5 and 10/7. I consider 1/1 to be more resolved (and therefore more important) than 2/1 and 7/5 goes better with 1/1 than 10/7 so I'm choosing 7/5.

Between 3/2 and 5/3 I have...

3/2	1.6667	701.9550
20/13	0.2538	745.7861
17/11	0.2995	753.6375
14/9	0.3651	764.9159
11/7	0.4675	782.4920
19/12	0.2719	795.5580
8/5	0.6500	813.6863
13/8	0.4038	840.5277
18/11	0.2929	852.5921
5/3	1.0667	884.3587

8/5 is the obvious choice here so 8/5 is in.

I now have...

1/1, __, __, 6/5, 5/4, 4/3, 7/5, 3/2, 8/5, 5/3, __, __, 2/1.

At this point I'm changing tactics: I want the I, IV and V chords available. IOW if the tonic is 1/1 I want a major chord on 1/1 and 4/3 and 3/2. If the tonic is E I want the E major, A major and B major chords available. Using the 9 notes listed just above, the E major and A major chords are available but not the B major. If I add 9/8 and 15/8 to the tuning then the B major chord becomes available. IOW the I, IV and V chords are available. I think that the I, IV, V chord progression (in

various combinations) is the most common progression in western music. So 9/8 and 15/8 are in and they both pair well melodically with 1/1 and 2/1. I now have...

1/1, __, 9/8, 6/5, 5/4, 4/3, 7/5, 3/2, 8/5, 5/3, __, 15/8, 2/1.

Between 1/1 and 9/8 I have...

1/1	4	0.0000
16/15	0.2583	111.7313
15/14	0.2762	119.4428
14/13	0.2967	128.2982
13/12	0.3205	138.5727
12/11	0.3485	150.6371
11/10	0.3818	165.0042
10/9	0.4222	182.4037
9/8	0.4722	203.9100

10/9 is too close to 9/8 to be implemented on a fretted instrument (the frets would be too close together for comfortable playing. 11/10, 12/11, 13/12 and 14/13 contain primes 11 or 13 and therefore will not pair nicely with the other notes so far. 15/14 does not pair well with 2/1. 16/15 is good, 2/1 over 16/15 gives 15/8, a medium strength melodic interval. So I'm choosing 16/15.

Between 5/3 and 15/8 I have...

5/3	1.0667	884.3587
17/10	0.3176	918.6417
12/7	0.4524	933.1291
19/11	0.2871	946.1951
7/4	0.7857	968.8259
16/9	0.3472	996.0900
9/5	0.6222	1017.5963
20/11	0.2818	1034.9958
11/6	0.5152	1049.3629
13/7	0.4396	1071.7018
15/8	0.3833	1088.2687

The likely candidates here are 12/7, 7/4, 16/9, 9/5 and 11/6. 12/7 is too close to 5/3. 7/4 is okay but being a 7 limit note will not pair well with many other notes. 16/9 is okay too but not as strong as 9/5. 11/6 has prime 11 and will not pair well with many other notes. Using 9/5 introduces another major chord on the 6/5 note so I'm choosing 9/5 which also pairs nicely with 2/1. I now have a 12 note per octave just scale which I call Eagle J.I....

1/1, 16/15, 9/8, 6/5, 5/4, 4/3, 7/5, 3/2, 8/5, 5/3, 9/5, 15/8, 2/1.

There is no room for additional notes as this would not be suitable for small fretted instruments like a ukelele or mandolin (i.e. some frets would be too close to each other for comfortable playing). This just tuning is not new. Back in the 90s I read somewhere that it is called “Just with 7 limit tritone” and I do not know who worked it out first.

There is no strong chord on the twelfth note (15/8) so the scale needs to be tempered. I raised the second note, 16/15, by 1.6 cents (from 111.7c to 113.3c) so that it could function as a 15/14 as well (within 6.7758 cents accuracy). I also raised the seventh note, 7/5, by 1.6 cents (from 582.5c to 584.1c) so that a minor chord could be played on the twelfth note (15/8) where all of the intervals in the chord are within 6.7758 cents of just.

I don't think that this is the best tempering of the just tuning. In May or June of 2016 I posted this just tuning on one of the Facebook tuning groups and (as I said in the introduction) a prominent member of the group, Paul Erlich suggested “tempering out the marvel comma, 225/224”. I don’t fully understand what this means but another member of the group, Jake Freivald, picked up on Paul’s suggestion and thought that *quantising* the scale to an EDO that tempers out 225/224 might be good.

EDO stands for Equal Division of the Octave. The standard western tuning, 12 Tone Equal Temperament, could be called 12EDO because it divides the octave into 12 equal steps. 31EDO divides the octave into 31 equal steps. Jake proposed three EDOs that temper out 225/224 and these were 41EDO, 53EDO and 72EDO. Note that only 12 notes (these are within +/-6.7758 cents of the 12 notes in Eagle J.I.) in each EDO are used and the others are not used. In the end (on 22nd June, 2016) I chose the 12 note subset of 53EDO, as my favourite, or ultimate, tuning. This tuning is the most accurate (closest to Eagle J.I.)of the three suggested by Jake. I’m calling it Eagle 53, or just Eagle. A big thank you to Paul and Jake for their input towards the creation of Eagle 53.

By coincidence Eagle 53 is not new. It is a 53EDO version of both Euler’s Genus Diatonico-chromaticum and Ellis’ Duodene, both of which are identical to Eagle J.I. except that Euler and Ellis used 45/32 (590.2c) instead of 7/5 (582.5c) in their just scales. I do not know who was the first person to quantise Euler’s/Ellis’ just scale to 53EDO. Here is a quick overview of Eagle 53…

Eagle 53

E	1/1	0.0	just
F	16/15	113.2075	sharp of just by 1.5 cents
F#	9/8	203.7736	flat of just by 0.1 cents
G	6/5	316.9811	sharp of just by 1.3 cents
G#	5/4	384.9057	flat of just by 1.4 cents
A	4/3	498.1132	sharp of just by 0.07 cents
A#	7/5	588.6792	sharp of just by 6.2 cents
B	3/2	701.8868	flat of just by 0.07 cents
C	8/5	815.0943	sharp of just by 1.4 cents
C#	5/3	883.0189	flat of just by 1.3 cents
D	9/5	1018.8679	sharp of just by 1.3 cents
D#	15/8	1086.7925	flat of just by 1.5 cents
E	2/1	1200.0	just

0.0, 113.2075, 203.7736, 316.9811, 384.9057, 498.1132, 588.6792, 701.8868, 815.0943, 883.0189, 1018.8679, 1086.7925, 1200.0

My scale requirements...
1. Every note between 1/1 and 2/1, when paired with both 1/1 and 2/1 must make a high strength or medium strength melodic interval. My formula for the strength of a melodic interval is 2/x + 2/y. Any result between 4 and 1 is a high strength interval and a result between 1 and 0.25 is medium strength interval. Less than 0.25 is a low strength interval. Using 5/4 we get 2/5 +2/4 = 0.9, a medium strength melodic interval. I allow the notes between 1/1 and 2/1 to be tempered by up to +/- 6.7758 cents. Low strength intervals occur elsewhere, but never in an interval where one of the notes is 1/1 or 2/1.
2. The tuning must not have more than 12 notes per octave. With Eagle 53 there is no room for another note because on a small fretted instrument such as a mandolin or ukulele some frets would be too close to each other for comfortable playing.
3. There must be at least one good chord on each of the twelve notes. These chords are listed below.
4. These good chords must be playable on a fretted instrument such as a guitar. I play guitar.

Eagle 53 meets all these requirements. Here are the good chords…
A Major chord looks like 2:3:4:5:6:8
A minor chord looks like 10:15:20:24:30:40
4:5:8:10:16 or 5:8:10:16:20 occur on F#, A# and D.

E (1/1) 2:3:4:5:6:8 and 10:15:20:24:30:40 Major and Minor
F (16/15) 2:3:4:5:6:8 Major
F# (9/8) 4:5:8:10:16
G (6/5) 2:3:4:5:6:8 Major
G# (5/4) 10:15:20:24:30:40 Minor
A (4/3) 2:3:4:5:6:8 and 10:15:20:24:30:40 Major and Minor
A# (7/5) 7:9:12 and 15:20:24 and 21:28:36 and 5:8:10:16:20
B (3/2) 2:3:4:5:6:8 and 10:15:20:24:30:40 Major and Minor
C (8/5) 2:3:4:5:6:8 Major
C# (5/3) 10:15:20:24:30:40 Minor
D (9/5) 4:5:8:10:16
D# (15/8)10:15:20:24:30:40 Minor

The last time i checked it looked like all of these chords (except the ones on the tritone, A#) are accurate within 1.4 cents (i.e. all the intervals in these chords are within 1.4 cents accuracy of just). There are many more good chords available in Eagle 53, I have only listed the strongest chords here.

If the notes of Eagle 53 are named numerically (i.e. 1, 2, 3 etc. up to 12 then six of the twelve notes can be used as a tonic because they have both a fourth (4/3) and a fifth (3/2) above them. These six notes are: 1, 4, 5, 8, 9 and 12.
You can hear the chords in Eagle 53 on my web app, the address is...

www.johnsmusic7.com/eagle.html

John O'Sullivan 23rd May, 2017.