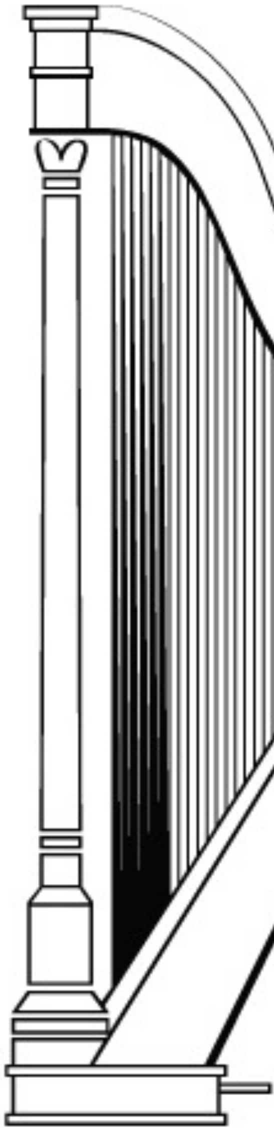
Nineteen Equal Divisions of the Octave



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Nineteen Equal Divisions of the Octave

Introduction

Back in 1995 I discovered that the standard Western musical tuning, 12 Tone Equal Temperament (12TET for short), is a compromise system where most of the intervals that occur are not perfectly in tune. An interval is the distance between two notes. Intervals can be expressed as ratios. An octave is 2/1. A perfect fifth is 3/2. A perfect fourth is 4/3. A major third is 5/4 and a minor third is 6/5.

In 12TET the octave is divided in to 12 equal steps. If you start with E the 12 notes are: E, F, F#, G, G#, A, A#, B, C, C#, D, D#. # means sharp, A# is the same note as Bb (B flat). The distance between any two adjacent notes is always the same: 100 cents. An octave (2/1) is 1200 cents wide. So if E is 0 cents, F is 100 cents, F# is 200 cents, G is 300 cents and so on. The problem is that most of the intervals that occur in 12TET are not perfectly in tune. In 12TET the octave (2/1) is perfectly in tune. A justly (just means pure) tuned 3/2 (perfect fifth) is 701.955 cents wide but in 12TET the 3/2 is tuned slightly narrower: 700 cents. So in 12TET the 3/2 is slightly out of tune by around 2 cents. This slight mistuning is barely noticeable but other intervals are much more out of tune in 12TET. 5/4 (a major third) is 13.7 cents wider than it should be (i.e. 400 cents, a just 5/4 is 386.3 cents). 6/5 (a minor third) is 15.6 cents narrower than it should be (i.e. 300 cents, a just 6/5 is 315.6 cents).

These latter intervals, 5/4 and 6/5, are much too out of tune for my liking. For years I would not use any interval that deviated from just by more then 6.7758 cents (256/255). I currently use a more relaxed tolerance: 8.474 cents (1024/1019).

When I discovered that 12TET is a compromise system I wondered if I could do better. Could there be a tuning better than 12TET? Over the years I came up with a few tunings that I thought were contenders: My Blue Temperament and later, Raven Temperament. Early on in my research I rediscovered Quarter Comma Meantone long before I found out it has been around for a long time and was used by some famous composers.

Finally in 2016 (with some assistance from Jake Freivald) I arrived at my Eagle 53 tuning. I know exactly what I want in a tuning and Eagle 53 ticks all the boxes. Other people want other things from a tuning and would therefore look elsewhere but for me Eagle 53 is top. I describe how I arrived at Eagle 53 in my book: Eagle 53 My Ultimate Musical Tuning (Third Edition). The book is available on Amazon.

Although Eagle 53 is my favorite tuning I am also interested in other tunings, in particular tunings that divide the octave into equal steps. These tunings are called EDOs (Equal Divisions of the Octave). 12 Tone Equal Temperament could also be called 12EDO. The most popular alternative EDOs include 17EDO, 19EDO, 22EDO, 24EDO and 31EDO. My favorite EDO (where *all* of the notes are used, not a subset, Eagle 53 is a 12 note subset of 53EDO) is 19EDO and I will explain why in this document.

Chapter One
Melody

I have a formula to determine how strong a just interval is. If the interval is x/y (where x and y are positive integers) the strength of the interval is 2/x + 2/y. I have reasons for not using 1/x + 1/y, this is explained in a few of my other books. If an interval has a strength value of 0.2 or higher then it is, for me, acceptable. This is in the context of melody, two notes played in sequence. In a harmony context, two notes played simultaneously, my cut off point is 0.4.

The narrowest just interval (apart from a unison, 1/1) that has a strength value >=0.2 is 20/19 (88.8 cents). Here is a comprehensive list of intervals that have a strength value of 0.2 or higher, over a one octave range. The ratios in the left column are the just intervals. The numbers in the middle column are the strength values using my formula and the numbers in the right hand column are the widths of the intervals in cents.

```
 1/1  - 4.0   -  0.0c
20/19 - 0.205 - 88.801c
19/18 - 0.216 - 93.603c
18/17 - 0.229 - 98.955c
17/16 - 0.243 - 104.96c
16/15 - 0.258 - 111.73c
15/14 - 0.276 - 119.44c
14/13 - 0.297 - 128.3c
13/12 - 0.321 - 138.57c
12/11 - 0.348 - 150.64c
11/10 - 0.382 - 165c
21/19 - 0.201 - 173.27c
10/9  - 0.422 - 182.4c
19/17 - 0.223 - 192.56c
 9/8  - 0.472 - 203.91c
17/15 - 0.251 - 216.69c
 8/7  - 0.536 - 231.17c
15/13 - 0.287 - 247.74c
 7/6  - 0.619 - 266.87c
20/17 - 0.218 - 281.36c
13/11 - 0.336 - 289.21c
19/16 - 0.23  - 297.51c
 6/5  - 0.733 - 315.64c
17/14 - 0.261 - 336.13c
11/9  - 0.404 - 347.41c
16/13 - 0.279 - 359.47c
21/17 - 0.213 - 365.83c
 5/4  - 0.9   - 386.31c
19/15 - 0.239 - 409.24c
14/11 - 0.325 - 417.51c
```

9/7 - 0.508 - 435.08c
22/17 - 0.209 - 446.36c
13/10 - 0.354 - 454.21c
17/13 - 0.271 - 464.43c
21/16 - 0.22 - 470.78c
4/3 - 1.17 - 498.04c
23/17 - 0.205 - 523.32c
19/14 - 0.248 - 528.69c
15/11 - 0.315 - 536.95c
11/8 - 0.432 - 551.32c
18/13 - 0.265 - 563.38c
7/5 - 0.686 - 582.51c
24/17 - 0.201 - 597c
17/12 - 0.284 - 603c
10/7 - 0.486 - 617.49c
23/16 - 0.212 - 628.27c
13/9 - 0.376 - 636.62c
16/11 - 0.307 - 648.68c
19/13 - 0.259 - 656.99c
22/15 - 0.224 - 663.05c
3/2 - 1.67 - 701.96c
23/15 - 0.22 - 740.01c
20/13 - 0.254 - 745.79c
17/11 - 0.299 - 753.64c
14/9 - 0.365 - 764.92c
25/16 - 0.205 - 772.63c
11/7 - 0.468 - 782.49c
19/12 - 0.272 - 795.56c
8/5 - 0.65 - 813.69c
21/13 - 0.249 - 830.25c
13/8 - 0.404 - 840.53c
18/11 - 0.293 - 852.59c
23/14 - 0.23 - 859.45c
5/3 - 1.07 - 884.36c
22/13 - 0.245 - 910.79c
17/10 - 0.318 - 918.64c
12/7 - 0.452 - 933.13c
19/11 - 0.287 - 946.2c
26/15 - 0.21 - 952.26c
7/4 - 0.786 - 968.83c
23/13 - 0.241 - 987.75c
16/9 - 0.347 - 996.09c
25/14 - 0.223 - 1003.8c
9/5 - 0.622 - 1017.6c
20/11 - 0.282 - 1035c
11/6 - 0.515 - 1049.4c
24/13 - 0.237 - 1061.4c
13/7 - 0.44 - 1071.7c
28/15 - 0.205 - 1080.6c
15/8 - 0.383 - 1088.3c
17/9 - 0.34 - 1101c
19/10 - 0.305 - 1111.2c

```
21/11 - 0.277 - 1119.5c
23/12 - 0.254 - 1126.3c
25/13 - 0.234 - 1132.1c
27/14 - 0.217 - 1137c
29/15 - 0.202 - 1141.3c
 2/1  - 3     - 1200c
```

I allow a maximum deviation from just of plus or minus 8.474 cents (1024/1019) so any interval within 8.474 cents of any just interval listed should be acceptable. Here are a few approximate ranges within which melodic intervals are, for me, illegal...

8.5c to 80.3c
479.3c to 489.5c
506.5c to 514.8c
671.5c to 693.5c
710.5c to 731.5c
1149.8c to 1191.5c

I'm just listing the larger ranges here, there are a few more narrower ones. Are there any EDOs where, over a one octave range, every note pairs nicely, melodically, with every other note?

1EDO is 0 cents and 1200 cents (1/1 and 2/1), a justly tuned octave. It's good but not very interesting due to scarcity of notes.

2EDO is 0c, 600c and 1200c. 600c is close to 17/12 so 2EDO is good but again, very few notes.

3EDO is 0c, 400c, 800c and 1200c. 400 cents is not within 8.474 cents of any good just interval listed above so 3EDO is out. So is 6EDO, 9EDO, 12EDO and 15EDO because these are all multiples of 3 which means they all contain a 400 cents interval which I deem to be illegal.

4EDO is 0c, 300c, 600c, 900c and 1200c. 900 cents is not within 8.474 cents of any good just interval listed above so 4EDO is out. So is 8EDO and 12EDO (again) because these are multiples of 4 which means they contain a 900 cents interval which I deem to be illegal.

5EDO contains a 720 cents interval which is not within 8.474 cents of any good interval listed above so 5EDO is out. 10EDO is out because 10 is a multiple of 5.

7EDO contains a 685.7 cents interval which is not within 8.474 cents of any melody interval I deem to be good. 14EDO is out as well because 14 is a multiple of 7.

11EDO contains a 872.7 cents interval which is not close to any of the good intervals in the list above so 11EDO is out.

13EDO is good! All the intervals over a one octave range are within 8.474 cents of a good interval in the list above.

```
  13EDO
   0.0c    1/1
  92.3c   19/18
 184.6c   10/9*
 276.9c   20/17
 369.2c   21/17
 461.5c   17/13
 553.8c   11/8*
 646.2c   16/11
 738.5c   23/15
 830.8c   21/13
 923.1c   17/10
1015.4c    9/5*
1107.7c   19/10
1200.0c    2/1
* indicates a good harmony interval.
```

EDOs from 15 to 141 (15EDO to 141EDO) are out because the narrowest interval that occurs (apart from a unison, 1/1) in these is between 8.5 cents and 80.3 cents, an illegal range.

So 13EDO has the property that, over a one octave range, every note pairs with every other note (in melody) in an acceptable manner. But it’s not a great tuning. There is no 3/2, 4/3, 5/3, 5/4, 6/5, 7/4, 7/5, 7/6, 8/5 or 8/7. There are only three intervals (not counting 1/1 and 2/1) that are useful in a harmony context and these are 10/9, 11/8 and 9/5. So for me 13EDO is a curiosity but not a practical tuning, especially if you want to play chords.

Chapter Two
Harmony

The strongest interval between 1/1 and 2/1 is the perfect fifth: 3/2. When choosing an EDO it would be nice if it had a perfect fifth within 8.474 cents of just. What EDOs have this property? We are looking for fifths between 693.481 cents and 710.429 cents. A just fifth (3/2) is 701.955 cents wide.

12EDO fifth is 700 cents
17EDO fifth is 705.882 cents
19EDO fifth is 694.737 cents
22EDO fifth is 709.091 cents
24EDO fifth is 700 cents
29EDO fifth is 703.448 cents
31EDO fifth is 696.774 cents
34EDO fifth is 705.882 cents

If a fifth (3/2) is good in an EDO then a fourth (4/3) will be good as well. The distance from 3/2 to 2/1 is 4/3.

Why stop at 34? I play guitar and I don't like having frets bunched up close together. Personally I would not be comfortable with more than 22 frets per octave but I know of other guitarists who are happy to play guitars with up to 34 frets per octave.

I like major chords. A six-note major chord on a regular guitar looks like... 2:3:4:5:6:8. These numbers show how the frequencies of each note relate to each other (approximately). So the frequencies of the lowest note and the note just above it make a ratio of roughly 2:3. The two highest notes make a ratio of 6:8 (approximately) which can be simplified to 3:4.

In general the smaller the numbers, the sweeter the chord. A six-note minor chord looks like... 10:15:20:24:30:40. These are much bigger numbers than the numbers in the major chord and indeed the minor chord sounds a lot weaker than the major chord.

I would like to have 2:3:4:5:6:8 major chords available in a tuning where all the intervals (pairs of notes) in the chord are within 8.474 cents of just. On a regular 12EDO guitar the 5 (a major third over the 4) in the 2:3:4:5:6:8 is too sharp by 13.7 cents, so it is too out of tune for my liking. Are 2:3:4:5:6:8 chords available in other EDOs within 8.474 cents of just? Yes, 19EDO, 31EDO and 34EDO have good major chords. I won't use the major chord in 22EDO because the 5/3 interval is too narrow by 11.63 cents.

What about 2:3:4:5:6:7 chords? The lowest EDO where these are within 8.474 cents of just is 31EDO. 31 frets per octave is too many for me.

The 2:3:4:5:6:8 chord sounds stronger than the 2:3:4:5:6:7 chord even though 7 is smaller than 8. The reason is that the 8 resonates strongly with the 2 and 4 (8:2 is a double octave and 8:4 is an octave). I prefer 2:3:4:5:6:8 chords to 2:3:4:5:6:7 chords.

If I had to choose an EDO, especially for guitars, it would be 19EDO. I'm not too concerned if I don't have any 7s available. 7/4, 7/5 and 7/6 don't occur in 12TET (or 12EDO, same thing) so I'm not really losing anything by switching to 19EDO. And major and minor chords are good in 19EDO.

I would consider using 22EDO and 17EDO as well. 22EDO has some interesting chords and 17EDO is good melodically. I don't see much advantage in 24EDO, the 12 extra notes don't get you many new good intervals. 11/9, 11/8 and 11/6 occur in 24EDO. I won't use 11/6 in a chord because the first overtone of the 6 beats against the fundamental of the 11.

Chapter Three
Scales and Chord Groups

So of the three EDOs I would consider (17, 19 and 22), which are the best for melodies (not chords)? Looking at 22EDO first, there are three intervals in 22EDO that I won't use in a melody context (over a one octave range). These are: 54.55c (one step), 327.273c (six steps) and 872.727c (sixteen steps). These intervals are not within 8.474 cents of any of the good intervals listed in chapter one. So if I am composing a melody I have to avoid notes that are one step apart, six steps apart and sixteen steps apart. This limits my options when composing melodies.

What about 17EDO? All the melodic intervals over a one octave range are acceptable except for the single step (70.588c). However, many of the stronger intervals are missing. 3/2 and 4/3 are good but there is no good 6/5, 5/4, 8/5, 5/3 or 9/5 in 17EDO.

How does 19EDO fare? Very good. Like 17EDO there is only one sour interval over a one octave range and that is the single step (63.158c). Also 6/5, 5/4, 4/3, 3/2, 8/5, 5/3 and 9/5 are all within 8.474 cents of just in 19EDO. So of the three EDOs (17, 19 and 22) 19 wins in both harmony (good major and minor chords in 19EDO) and melody (only one sour interval between 1/1 and 2/1 and all of the strongest intervals occur).

24EDO has four intervals (over an octave) that I deem to be illegal: 50c, 400c, 900c and 1150c. 31EDO has three intervals (over an octave) that I deem to be illegal: 38.71c, 77.42c and 1161.29c.

Below is a list of intervals that occur in 19EDO over a five-octave range. Melodically they are all good except for the 63 cents single step. Intervals that are good in a harmony context are indicated by an asterisk *.

```
 1   0
 2  63.1579  NO
 3  126.316  15/14 and 14/13
 4  189.474  10/9* and 19/17
 5  252.632  15/13
 6  315.789  6/5*
 7  378.947  5/4*
 8  442.105  9/7* and 22/17
 9  505.263  4/3*
10  568.421  18/13
11  631.579  23/16 and 13/9
12  694.737  3/2*
13  757.895  17/11 and 14/9
14  821.053  8/5*
15  884.211  5/3*
16  947.368  19/11 and 26/15
```

17 1010.53   25/14 and 9/5*
18 1073.68   13/7 and 28/15
19 1136.84   25/13 and 27/14 and 29/15
20 1200   2/1*
1263.16 29/14 and 27/13 and 25/12
1326.32 15/7 and 28/13
1389.47 20/9 and 29/13
1452.63 30/13
1515.79 12/5*
1578.95 5/2*
1642.11 18/7 and 31/12
1705.26 8/3*
1768.42 36/13 and 25/9
1831.58 23/8 and 26/9
1894.74 3/1*
1957.89 34/11 and 31/10 and 28/9
2021.05 16/5* and 29/9
2084.21 10/3*
2147.37 31/9 and 38/11
2210.53 25/7 and 43/12 and 18/5*
2273.68 26/7 and 41/11
2336.84 27/7 and 31/8
2400   4/1*
2463.16 29/7 and 25/6*
2526.32 30/7 and 43/10
2589.47 40/9 and 49/11
2652.63 37/8 and 51/11
2715.79 43/9 and 24/5* and 53/11
2778.95 5/1*
2842.11 36/7 and 31/6 and 57/11
2905.26 16/3* and 59/11 and 43/8
2968.42 61/11 and 50/9 and 39/7
3031.58 23/4* and 52/9
3094.74 6/1*
3157.89 68/11 and 31/5* and 56/9
3221.05 32/5* and 45/7 and 58/9 and 71/11
3284.21 73/11 and 20/3*
3347.37 62/9 and 69/10 and 76/11
3410.53 50/7 and 43/6 and 79/11 and 36/5*
3473.68 52/7 and 67/9 and 82/11
3536.84 77/10 and 54/7 and 85/11 and 31/4*
3600   8/1*
For the remainder of this list I'm just mentioning one ratio per interval even though other close good ratios occur.
3663.16 25/3*
3726.32 43/5*
3789.47 98/11
3852.63 37/4*
3915.79 48/5*
3978.95 10/1*
4042.11 31/3*
4105.26 43/4*

4168.42 100/9
4231.58 23/2*
4294.74 12/1*
4357.89 62/5*
4421.05 64/5*
4484.21 40/3*
4547.37 69/5*
4610.53 43/3*
4673.68 119/8
4736.84 31/2*
4800   16/1*
4863.16 83/5*
4926.32 86/5*
4989.47 89/5*
5052.63 37/2*
5115.79 96/5*
5178.95 20/1*
5242.11 62/3*
5305.26 107/5*
5368.42 111/5*
5431.58 23/1*
5494.74 24/1*
5557.89 99/4*
5621.05 77/3*
5684.21 80/3*
5747.37 83/3*
5810.53 86/3*
5873.68 119/4*
5936.84 154/5*
6000   32/1*

Any melodic interval wider than 6,000 cents I deem to be acceptable. In the table below nine scales are listed. The first has ten notes: 1/1 and 2/1 and eight notes in between (nine notes per octave). The other eight scales have nine notes: 1/1 and 2/1 and seven notes in between (eight notes per octave). All of these scales have a 1/1, 4/3, 3/2 and 2/1. There are no two notes a single step apart which is, for me, a sour melodic interval.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
©		©		©		©		©			©		©		©		©		©
©		©		©		©		©			©		©			©			©
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1/1	x	15/14	10/9	15/13	6/5	5/4	9/7	4/3	18/13	13/9	3/2	14/9	8/5	5/3	19/11	9/5	13/7	25/13	2/1

I don’t like having three or more notes bunched up close together, two is fine but three or more, no (because 15/14 is tolerable but not very strong and the fewer 15/14s the better). If ‘1’ is a note and ‘0’ is a space then I don’t like 10101 (in 19 EDO). 101001 is okay and so is 100101 but not 10101. Below are 16 eight-note scales (seven notes per octave) that do not have any notes that follow the 10101 pattern (over a one octave range).

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
©		©			©			©			©		©			©			©
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1/1	x	15/14	10/9	15/13	6/5	5/4	9/7	4/3	18/13	13/9	3/2	14/9	8/5	5/3	19/11	9/5	13/7	25/13	2/1

I am also not overly fond of any two notes that are four steps apart (252.6 cents or 15/13). This interval is certainly tolerable, but it is weak, and it would be nice if we could avoid it. Of the sixteen scales in the table above the following seven scales have this four-step interval: numbers 2, 5, 6, 7, 8, 10 and 14. So the other nine scales are the best: 1, 3, 4, 9, 11, 12, 13, 15 and 16.

Finally, if we want to use a scale that accommodates the I, IV and V chords (e.g. E major, A major and B major) then scale number 16 in the grid above is not only good, but for me, the best. The notes used are... 1, 4, 7, 9, 12, 15, 18 and 20. Or 1/1, 10/9, 5/4, 4/3, 3/2, 5/3, 13/7 and 2/1. The 13/7 functions as a 5/4 over the 3/2.

Whatever scale you choose, regardless of whether it has nine, eight or seven notes per octave, any chord progression should sound good if you stick to the notes in the scale (could be over a one octave range or many octaves). So ask yourself: can I get a major chord on this note using only the notes in this scale? If not, how about a 2:3:4:6 chord? If not, how about a minor chord? Or a 4:5:8:10 chord? As a general rule of thumb, if you avoid notes that are only one 63 cents step apart you can’t play a sour note (melodically) in 19EDO.

Clearly rooted chord progressions

Here are some chord groups that have a very clear, or strong, root chord). The ideal chords for these chord groups are major chords because they are also strongly rooted (they contain a 1, 2, 4, 8 or 16). A six-note major chord is 2:3:4:5:6:8. Four-note chords could work as well (e.g. 2:3:4:5 or 4:5:6:8). The idea is that the pitches of the lowest notes in the major chords used should correspond to any of the following proportions (within +/- 8.474 cents accuracy)...

3:4:5:6

4:5:6:8

5:6:8:10

So with the 3:4:5:6 chord group, starting on the lowest note (index 1), the major chords should be played on indices 1, 9, 15 and 20. The chord on 4 (in the 3:4:5:6 group) is the root chord (on index 9) because 4 is a power of 2 and therefore sounds resolved.

With the 4:5:6:8 chord group, starting on index 1, the major chords should be played on indices 1, 7, 12 and 20. The chords on 4 or 8 (in the 4:5:6:8 group) function as root chords (on indices 1 or 20) because 4 and 8 are powers of 2.

With the 5:6:8:10 chord group, starting on index 1, the major chords should be played on indices 1, 6, 14 and 20. The chord on 8 (in the 5:6:8:10 group) is the root chord (on index 14) because 8 is a power of 2. I have written about this power of 2 idea in some of my recent books.

So the root chords should be points of rest or resolution.

I'm starting all of these on index 1 but obviously you could choose any number you like and scale the other numbers up accordingly.

Proportions (chord groups) that don't contain a 1, 2, 4, 8 or 16 are, for me, not strongly rooted (or don't have a clear tonic), they have no clear point of rest or resolution.

Chapter Four
Keyboards and Guitars

I'm not sure about the best layout for piano style instruments or keyboards implementing 19EDO. I considered ten white keys and nine black keys per octave but some chords would be unplayable because the relevant keys would be too far apart. I'll leave it to the reader to investigate a suitable key arrangement. There are several tuning groups on Facebook and you could ask there.

You could choose to use only the notes found in the nine, eight or seven-note scales I described earlier. If you go this route you could retune the relevant notes on a regular electric retunable keyboard. This way three, four or five notes (per octave) would not be used. Some calculations should reveal the available chords. The strongest chords to look for are major (2:3:4:5:6:8 or a subset), minor (10:15:20:24:30:40 or 10:12:15:20), 2:3:4:6:8 and 4:5:8:10:16.

For guitarists here are specifications for fret placements on a 19EDO guitar. Fret number 1 is the fret closest to the nut. The 's' stands for 'scale length' which is usually the distance from the nut to the saddle on the lightest string. The saddle is usually angled slightly so that the distance from nut to saddle is slightly greater for the thicker strings. So if the scale length is 66cm (around 26 inches) the distance from the nut to the first fret is 66cm x 0.035824 = 2.36cm. The distance from the nut to the second fret is 66cm x 0.0703646 = 4.64cm. And so on.

You could buy a cheap guitar, rip out the frets, fill the slots with resin or wood cement, cut new slots according to the list below and hammer in new frets. I've done this a few times for various tunings and the resulting fretboards were usable but I recommend hiring a luthier to do the job if you can afford it.

Fret 1 = 0.035824s
Fret 2 = 0.0703646s
Fret 3 = 0.103668s
Fret 4 = 0.135778s
Fret 5 = 0.166738s
Fret 6 = 0.196589s
Fret 7 = 0.22537s
Fret 8 = 0.253121s
Fret 9 = 0.279877s
Fret 10 = 0.305674s
Fret 11 = 0.330548s
Fret 12 = 0.35453s
Fret 13 = 0.377654s
Fret 14 = 0.399949s

Fret 15 = 0.421445s
Fret 16 = 0.442171s
Fret 17 = 0.462155s
Fret 18 = 0.481422s
Fret 19 = 0.5s
Fret 20 = 0.517912s
Fret 21 = 0.535182s
Fret 22 = 0.551834s
Fret 23 = 0.567889s
Fret 24 = 0.583369s
Fret 25 = 0.598294s
Fret 26 = 0.612685s
Fret 27 = 0.62656s
Fret 28 = 0.639938s
Fret 29 = 0.652837s
Fret 30 = 0.665274s
Fret 31 = 0.677265s
Fret 32 = 0.688827s
Fret 33 = 0.699974s
Fret 34 = 0.710722s
Fret 35 = 0.721086s
Fret 36 = 0.731077s
Fret 37 = 0.740711s
Fret 38 = 0.75s

How should the guitar be tuned? For me the ideal tuning from lowest (bass) string to highest (treble) would be indices 1, 10, 18, 25, 31 and 39. This translates to 0c (1/1), 568.4c (18/13), 1073.7c (13/7), 1515.8c (12/5), 1894.7c (3/1) and 2400.0c (4/1). I chose this tuning so that open E major shape barre chords (2:3:4:5:6:8) and open E minor shape barre chords (10:15:20:24:30:40) are playable and look identical to their 12TET counterparts except that the frets are a bit closer together.

Again, depending on the scale you choose the chords to look for are 2:3:4:5:6:8 or 2:3:4:5 or 4:5:6:8 (major chords) or 10:15:20:24:30:40 or 10:15:20:24 (minor chords) or 2:3:4:6 or 4:5:8:10. I call these latter chords “two-tone” chords because they contain only two pitch classes. Some musicians won’t use these two-tone chords but I like them.

Afterword

This PDF is free, share copies of it with whoever you like but I own the copyright on most of the text (you can't copyright or patent mathematics or numbers). I currently have seven books in print and these are...

Eagle 53 My Ultimate Musical Tuning (Third Edition)... The book describes how I arrived at my Eagle 53 tuning.

The Eagle 53 Pianist... 96 scales and 2,750 lush chords are described for pianos/keyboards.

Eagle 53 Jazz Chords... 2,999 non-lush (jazz) chords for pianos/keyboards.

John's Rules Music... a very short book about some rules that I worked out and follow when I am composing. These rules could apply to any tuning. There are only 19 pages of text. The other 80 pages list 96 scales found in Eagle 53 and 2196 just intervals that I consider to be acceptable over a six octave range.

The Eagle 53 Guitarist Lush Chords... 96 scales and diagrams of 940 lush chords that are playable on guitars fretted for Eagle 53. A lush chord is both strong and clearly rooted. A jazz chord is weaker and/or is not clearly rooted.

The Eagle 53 Guitarist Jazz Chords... diagrams of 823 jazz chords on Eagle 53 guitars.

The Arabian Scale in Eagle 53... The book is a 'proof of concept'. 507 chords are listed (for both Eagle 53 guitars and keyboards) and if I'm right any combination of any of these 507 chords, played in any order, should sound sweet. At the time of writing, a free PDF of this book is available for a limited time... www.johnsmusic7.com/ArabianEagle53.pdf

These books are available on Amazon, check out the links on my web site. There are videos, MP3s, a web app and more info there as well...

www.johnsmusic7.com

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27th February, 2021