# Beatless Chords - ©John O'Sullivan - 28th July, 2021

I have a formula for the consonance, or concordance, of a musical interval n/d where n and d are positive integers. The formula is 2/n + 2/d. If an interval has a value (using this formula) equal to or greater than 0.4 then, I thought, it should be good (in a harmony context, the cut-off is 0.2 in a melody context). 'n' is numerator and 'd' is denominator.

However, I could hear some noticeable beating in the 15/7 interval.
2/15 + 2/7 = 0.419 which is greater than my cut-off point of 0.4.
I usually associated beating with narrow intervals such as a semitone (100 cents) but 15/7 is a wide interval (1319 cents). It seems that the beating I could hear was between the fundamental (first harmonic) of the 15 and the first overtone (or second harmonic) of the 7 (which is 14). 15/14 is a very narrow interval and hence the beating.

How narrow should an interval be before the beating (which I find unpleasant) becomes intolerable? Using my 2/n + 2/d formula the narrowest interval (apart from a unison, 1/1) with a value >=0.4 is 10/9 (182.4 cents). I allow a tempering of an interval of up to +/- 8.5 cents. 182.4 - 8.5 = 173.9 cents. So I decided that intolerable beating should start where concordance ends: any harmony interval between 8.5 cents (a tempered unison) and 173.9 cents (a tempered 10/9) should be illegal.

Over a two octave range I identified five intervals where the fundamental of the higher note and the first overtone of the lower note made an interval narrower than 173.9 cents. These are... 11/6, 13/7, 15/7, 13/6 and 11/5. So for me these intervals are illegal in any chord due to significant beating which I don't like.

What about the third and subsequent harmonics? (or the second and subsequent overtones?). My current understanding is that these are too faint (low intensity) to matter. Overtones get progressively fainter (quieter) as you progress along the harmonic series. So any beating among the third and subsequent harmonics is for me, currently, insignificant.

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Here is something I have been wrestling with for two or more years. I don't like minor thirds (6/5, 315.6 cents) in a harmony (not melody) context. I can hear significant beating in 6/5 intervals on my Eagle fretted guitar. All minor chords contain a 6/5. And I don't like minor chords much either, I was never a big fan of them. I like 5/4 (386.3 cents). And So if I were to be very strict I would 'ban' any intervals (except for unisons) narrower than 377.8 cents (a just 5/4) tempered narrower by 8.5 cents.

This means that all minor chords and 2:3:4:5:6:8 major chords (contain a 5:6) would be illegal. And look at the 5/3 interval. The fundamental of the 5 and the first overtone of the 3 (which is a 6) makes a 6/5 interval, which I don't like. So, again, if I were to be very strict, I wouldn't allow a 5/3 interval (major sixth) which almost every person reading this will find preposterous.

But, I am not always very strict and I am happy to use 6/5 and 5/3 and minor chords sometimes when I am composing.

On the other hand, if I *were* to be very strict and rule out intervals narrower than 377.8 cents, or intervals where the fundamental of the higher note paired with the first overtone of the lower note makes an interval between 8.5c and 377.8c what chords would be available?

Not many. But on eleven of the twelve notes in Eagle 53 at least one very strong and strongly rooted chord occurs where all the intervals in the chord are within 1.4 cents of just. The chord on A# (7/5) is the exception.

On a piano keyboard tuned to Eagle 53, where E is on 1/1 or 0 cents, here are the strongest available chords...

E (1/1) 2:3:4:6:8
F (16/16) 2:3:4:6:8
F# (9/8) 2:4:5:8 or 4:5:8:10:16
G (6/5) 2:3:4:6:8
G# (5/4) 2:3:4:6:8
A (4/3) 2:3:4:6:8
A# (7/5) 7:10:14:20:28 (20/7 ≈ 17/6)
B (3/2) 2:3:4:6:8
C (8/5) 2:3:4:6:8

C# (5/3) 2:3:4:6:8
D (9/5) 2:4:5:8 or 4:5:8:10:16
D# (15/8) 2:3:4:6:8

On a guitar fretted for Eagle 53 and E is on 1/1...

E (1/1) 2:3:4 or 2:3:4:6
F (16/16) 2:3:4
F# (9/8) 4:5:8:10
G (6/5) 2:3:4
G# (5/4) 2:3:4
A (4/3) 2:3:4
A# (7/5) 7:10:14:20 (20/7 ≈ 17/6)
B (3/2) 2:3:4
C (8/5) 2:3:4
C# (5/3) 2:3:4
D (9/5) 2:4:5:8 or 4:5:8:10
D# (15/8) 2:3:4

2:3:4:6 occurs on F, G, G#, A, B, C, C# and D# but these are very difficult to play on an Eagle fretted guitar. If I allowed 3:5 I would still prefer 2:3:4:6:8 chords to 2:3:4:5:8 chords because the 6 resonates more with the other notes than the 5.

If I want more variety of chords I have to allow some mild beating. My books for Eagle tuned keyboards and Eagle fretted guitars list several thousand chords. These books are described on my website...

www.johnmusic7.com

Below is a list of harmony intervals with a strength value >=0.4 using my formula. The left column lists the just intervals. The next column shows the widths of the intervals in cents. The numbers to the right of these are the strength values using the formula. The five intervals marked with --- I never use (in harmony). The intervals marked with ### I avoid only *if* I am being very strict (melodically all these intervals are, for me, good).

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1/1 = 0.0 cents 4.00
10/9 = 182.404 cents 0.42  ###
9/8 = 203.91 cents 0.47    ###
8/7 = 231.174 cents 0.54   ###
7/6 = 266.871 cents 0.62   ###
6/5 = 315.641 cents 0.73   ###
11/9 = 347.408 cents 0.40  ###
5/4 = 386.314 cents 0.90
9/7 = 435.084 cents 0.51
4/3 = 498.045 cents 1.17
11/8 = 551.318 cents 0.43
7/5 = 582.512 cents 0.69
10/7 = 617.488 cents 0.49
3/2 = 701.955 cents 1.67
11/7 = 782.492 cents 0.47
8/5 = 813.686 cents 0.65
13/8 = 840.528 cents 0.40  ###
5/3 = 884.359 cents 1.07   ###
12/7 = 933.129 cents 0.45  ###
7/4 = 968.826 cents 0.79   ###
9/5 = 1017.6 cents 0.62    ###
11/6 = 1049.36 cents 0.52  ---
13/7 = 1071.7 cents 0.44   ---
2/1 = 1200 cents 3.00
15/7 = 1319.44 cents 0.42  ---
13/6 = 1338.57 cents 0.49  ---
11/5 = 1365 cents 0.58     ---
9/4 = 1403.91 cents 0.72   ###
16/7 = 1431.17 cents 0.41  ###
7/3 = 1466.87 cents 0.95   ###
12/5 = 1515.64 cents 0.57  ###
17/7 = 1536.13 cents 0.40  ###
5/2 = 1586.31 cents 1.40
13/5 = 1654.21 cents 0.55
8/3 = 1698.04 cents 0.92
11/4 = 1751.32 cents 0.68
14/5 = 1782.51 cents 0.54
17/6 = 1803 cents 0.45
3/1 = 1901.96 cents 2.67
19/6 = 1995.56 cents 0.44
16/5 = 2013.69 cents 0.53
13/4 = 2040.53 cents 0.65
10/3 = 2084.36 cents 0.87
17/5 = 2118.64 cents 0.52
7/2 = 2168.83 cents 1.29
18/5 = 2217.6 cents 0.51
11/3 = 2249.36 cents 0.85
15/4 = 2288.27 cents 0.63
19/5 = 2311.2 cents 0.51
23/6 = 2326.32 cents 0.42
4/1 = 2400 cents 2.50
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