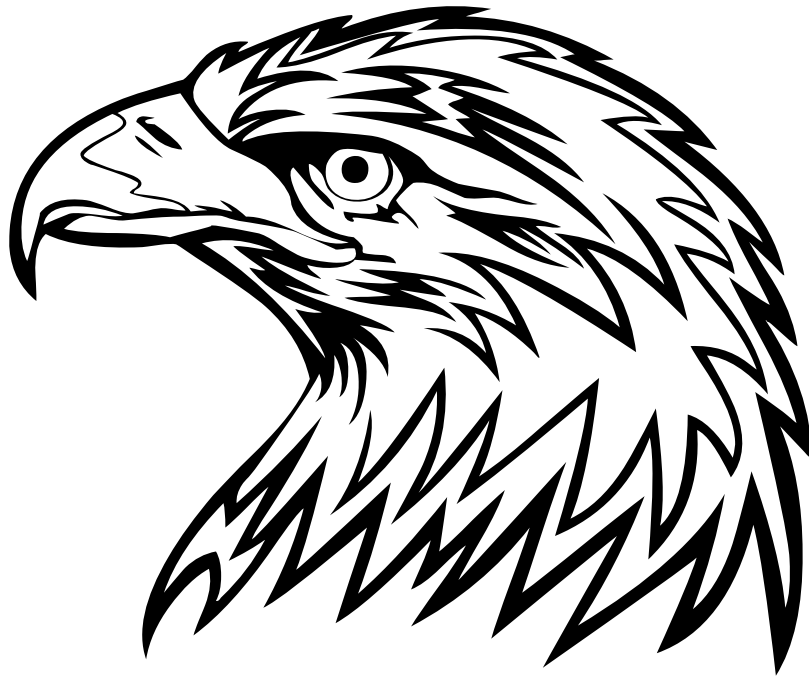


John's Just Intonation



John O'Sullivan

©John O'Sullivan 2024

INTRODUCTION

I have a formula for quantifying the strength values of just intervals. For any just interval n/d (n = numerator and d = denominator and n and d are positive integers) the strength value of the interval is $2/n + 2/d$. Why not $1/n + 1/d$? See page 4 of my book: John's Rules Music. You can download this book free at...

<https://www.johnsmusic7.com/JohnsRulesMusic.pdf>

It seems to me that if the strength value of a just interval (using the $2/n + 2/d$ formula) is equal to, or greater than, 0.2 then the interval is good in a melody (notes played in sequence), not harmony (notes played simultaneously) context.

And it seems to me that if the strength value is equal to, or greater than, 0.4 then the interval is good in a harmony (notes played simultaneously) context. There are five harmony intervals that are exceptions to this, see below.

I consider my approach to just intonation to be "strict" just intonation and I'm calling it "John's Just Intonation".

MELODY

Here is a complete list of all good melodic intervals (over a one octave range) that have a strength value greater than or equal to 0.2 using my formula. The left column shows the just intervals, the middle column shows the strength values and the right column shows the widths of the intervals in cents...

1/1 - 4.0 - 0c
20/19 - 0.205 - 88.801c
19/18 - 0.216 - 93.603c
18/17 - 0.229 - 98.955c
17/16 - 0.243 - 104.96c
16/15 - 0.258 - 111.73c
15/14 - 0.276 - 119.44c
14/13 - 0.297 - 128.3c
13/12 - 0.321 - 138.57c
12/11 - 0.348 - 150.64c
11/10 - 0.382 - 165c
21/19 - 0.201 - 173.27c
10/9 - 0.422 - 182.4c
19/17 - 0.223 - 192.56c
9/8 - 0.472 - 203.91c
17/15 - 0.251 - 216.69c
8/7 - 0.536 - 231.17c
15/13 - 0.287 - 247.74c
7/6 - 0.619 - 266.87c
20/17 - 0.218 - 281.36c
13/11 - 0.336 - 289.21c
19/16 - 0.23 - 297.51c
6/5 - 0.733 - 315.64c
17/14 - 0.261 - 336.13c
11/9 - 0.404 - 347.41c
16/13 - 0.279 - 359.47c
21/17 - 0.213 - 365.83c
5/4 - 0.9 - 386.31c
19/15 - 0.239 - 409.24c
14/11 - 0.325 - 417.51c
9/7 - 0.508 - 435.08c

22/17 - 0.209 - 446.36c
13/10 - 0.354 - 454.21c
17/13 - 0.271 - 464.43c
21/16 - 0.22 - 470.78c
4/3 - 1.17 - 498.04c
23/17 - 0.205 - 523.32c
19/14 - 0.248 - 528.69c
15/11 - 0.315 - 536.95c
11/8 - 0.432 - 551.32c
18/13 - 0.265 - 563.38c
7/5 - 0.686 - 582.51c
24/17 - 0.201 - 597c
17/12 - 0.284 - 603c
10/7 - 0.486 - 617.49c
23/16 - 0.212 - 628.27c
13/9 - 0.376 - 636.62c
16/11 - 0.307 - 648.68c
19/13 - 0.259 - 656.99c
22/15 - 0.224 - 663.05c
3/2 - 1.67 - 701.96c
23/15 - 0.22 - 740.01c
20/13 - 0.254 - 745.79c
17/11 - 0.299 - 753.64c
14/9 - 0.365 - 764.92c
25/16 - 0.205 - 772.63c
11/7 - 0.468 - 782.49c
19/12 - 0.272 - 795.56c
8/5 - 0.65 - 813.69c
21/13 - 0.249 - 830.25c
13/8 - 0.404 - 840.53c
18/11 - 0.293 - 852.59c
23/14 - 0.23 - 859.45c
5/3 - 1.07 - 884.36c
22/13 - 0.245 - 910.79c
17/10 - 0.318 - 918.64c
12/7 - 0.452 - 933.13c
19/11 - 0.287 - 946.2c
26/15 - 0.21 - 952.26c

7/4 - 0.786 - 968.83c
23/13 - 0.241 - 987.75c
16/9 - 0.347 - 996.09c
25/14 - 0.223 - 1003.8c
9/5 - 0.622 - 1017.6c
20/11 - 0.282 - 1035c
11/6 - 0.515 - 1049.4c
24/13 - 0.237 - 1061.4c
13/7 - 0.44 - 1071.7c
28/15 - 0.205 - 1080.6c
15/8 - 0.383 - 1088.3c
17/9 - 0.34 - 1101c
19/10 - 0.305 - 1111.2c
21/11 - 0.277 - 1119.5c
23/12 - 0.254 - 1126.3c
25/13 - 0.234 - 1132.1c
27/14 - 0.217 - 1137c
29/15 - 0.202 - 1141.3c
2/1 - 3 - 1200c

Look at this sequence of numbers...

1:2:3:4:5:6:7:8:9:10:11:12:13:14:15:16:17:18:19:20

Any two numbers from the twenty listed above will correspond to a good (for me) melodic interval (e.g. 19/17 is a good melodic interval).

If the all of the frequencies of the notes you choose are proportional to some or all of the twenty numbers above then it is impossible to play a melody that is sour or out of tune. The intervals 21/20 or 22/21 or narrower (apart from a unison) are too weak for my taste.

Below is a list of “good” just scales that span one octave. The scales with fewer notes are stronger but don’t have much variety. The scales with more notes are weaker but have more variety.

A tonic note is a “root” note or point of resolution or point of rest. If a 4 or 8 or 16 occurs in the scales below then these are tonics or “root” notes. Root notes are in **bold** below.

3:4:5:6

4:5:6:7:8

5:6:7:8:9:10

6:7:8:9:10:11:12

7:8:9:10:11:12:13:14

8:9:10:11:12:13:14:15:**16**

9:10:11:12:13:14:15:**16**:17:18

10:11:12:13:14:15:**16**:17:18:19:20

If we allow tempering of intervals (my tempering tolerance is not more than 8.474 cents ($1024/1019$) from just) then a scale with 14 notes spanning an octave (14 if you include both $1/1$ and $2/1$) is available. That scale is 13EDO (13 Equal Divisions of the Octave). Unless I’m mistaken, **all** of the intervals in 13EDO (over a one octave range) are within 8.474 cents of a ‘good’ melody interval in the list above but none of them (apart from the octave) are particularly strong. There are very few good harmony intervals in 13EDO.

HARMONY

Here is a list of the harmony intervals that I consider to be 'good' (with five exceptions, see below)...

1/1 - 4.0 - 0c
10/9 - 0.422 - 182.4c
9/8 - 0.472 - 203.91c
8/7 - 0.536 - 231.17c
7/6 - 0.619 - 266.87c
6/5 - 0.733 - 315.64c
11/9 - 0.404 - 347.41c
5/4 - 0.9 - 386.31c
9/7 - 0.508 - 435.08c
4/3 - 1.17 - 498.04c
11/8 - 0.432 - 551.32c
7/5 - 0.686 - 582.51c
10/7 - 0.486 - 617.49c
3/2 - 1.67 - 701.96c
11/7 - 0.468 - 782.49c
8/5 - 0.65 - 813.69c
13/8 - 0.404 - 840.53c
5/3 - 1.07 - 884.36c
12/7 - 0.452 - 933.13c
7/4 - 0.786 - 968.83c
9/5 - 0.622 - 1017.6c
11/6 - 0.515 - 1049.4c. beating
13/7 - 0.44 - 1071.7c beating
2/1 - 3 - 1200c
15/7 - 0.419 - 1319.4c beating
13/6 - 0.487 - 1338.6c beating
11/5 - 0.582 - 1365c beating
9/4 - 0.722 - 1403.9c
16/7 - 0.411 - 1431.2c
7/3 - 0.952 - 1466.9c
12/5 - 0.567 - 1515.6c
17/7 - 0.403 - 1536.1c
5/2 - 1.4 - 1586.3c

13/5 - 0.554 - 1654.2c
8/3 - 0.917 - 1698c
11/4 - 0.682 - 1751.3c
14/5 - 0.543 - 1782.5c
17/6 - 0.451 - 1803c
3/1 - 2.67 - 1902c
19/6 - 0.439 - 1995.6c
16/5 - 0.525 - 2013.7c
13/4 - 0.654 - 2040.5c
10/3 - 0.867 - 2084.4c
17/5 - 0.518 - 2118.6c
7/2 - 1.29 - 2168.8c
18/5 - 0.511 - 2217.6c
11/3 - 0.848 - 2249.4c
15/4 - 0.633 - 2288.3c
19/5 - 0.505 - 2311.2c
23/6 - 0.42 - 2326.3c
4/1 - 2.5 - 2400c

There are five intervals in this list (11/6, 13/7, 15/7, 13/6 and 11/5) that have strength values greater than 0.4 but sound dissonant. Using sine wave tones that have a fundamental but no overtones these intervals are acceptable but with instruments with a timbre that is close to a regular harmonic series (e.g. guitar strings) I can hear some significant unpleasant beating (see page 16 of my book: [John's Rules Music](#) (link above) which explains beating). It seems to me that with these five intervals the first overtone of the lower note beats significantly against the fundamental of the higher note. I won't use these intervals in a chord.

Many just scales contain a 45/32 interval which I won't use (it has a strength value of 0.107 which is below my cutoff point of 0.2). And yet it sounds okay in both a melody and harmony context. The reason it sounds okay is because it is only 7.7 cents away from a just 7/5. If I'm composing in strict just intonation I won't use a 45/32 interval.

If I am constructing just chords then every note should pair with every other note exactly according to any of the intervals (excluding the five exceptions) listed above. The list above spans only two octaves. See my book [John's Rules](#)

Music for a list of all 'good' melody and harmony intervals spanning six octaves.

Also, these just chords must have a strength value greater than or equal to 0.4. Look at the 10:12:15:20:24:30 minor chord. The strength value of this chord is...

$$2/10 + 2/12 + 2/15 + 2/20 + 2/24 + 2/30 = 0.6833$$

0.6883 is greater than 0.4 and so the chord is, for me at least, "good".

The method for quantifying the strength of chords is to write the chord as a sequence of integers and then add together the inverses of each integer multiplied by 2 for a total strength value. Anything ≥ 0.4 should indicate a good chord.

A just chord I like a lot is 2:3:4:5:7:9.

Why not use 6 and 8? Because I am not a lover of harmony intervals narrower than $5/4$ (apart from unisons). $6/5$, $7/6$, $8/7$ and $9/8$ are narrower than $5/4$.

The last time I checked it seemed that the 2:3:4:5:7:9 chord occurs on C and F in Eagle 53 (if E is $1/1$) and all of the intervals in this chord are within 6.2 cents of just.

AFTERWORD

So this is my take on Just Intonation. As I said this is “strict” just intonation and I’m calling it John’s Just Intonation. It is mine because it uses my $2/n + 2/d$ formula with cutoff points of 0.2 for melody and 0.4 for harmony.

Here’s the link to my book again...

<https://www.johnsmusic7.com/JohnsRulesMusic.pdf>

My website...

<https://www.johnsmusic7.com>

John O’Sullivan
11th October, 2024